TRIGONOMETRÍA

Profesor
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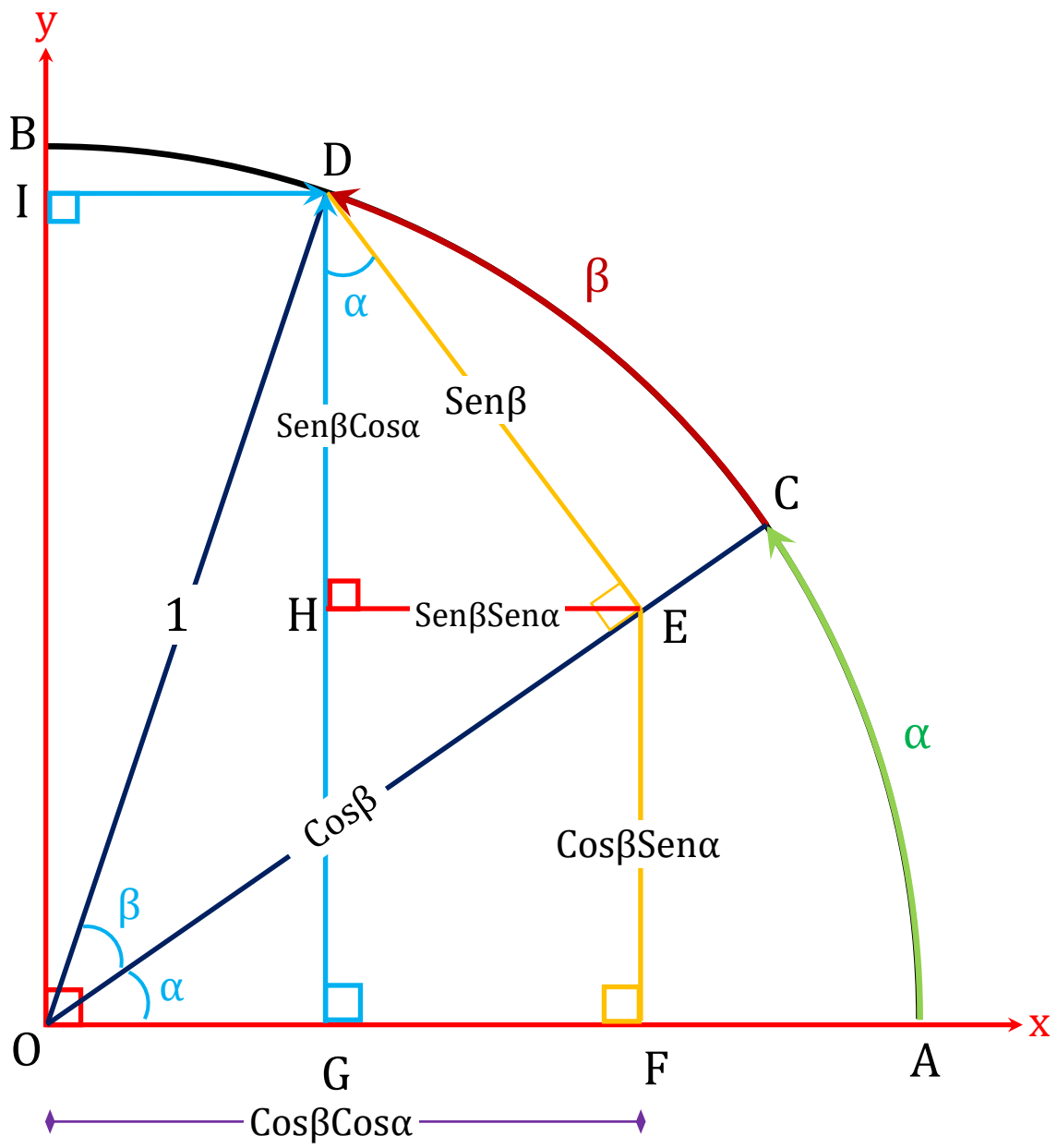
IDENTIDADES TRIGONOMÉTRICAS

IDENTIDADES TRIGONOMÉTRICAS PARA EL ÁNGULO COMPUESTO

IDENTIDADES PARA EL ÁNGULO COMPUESTO



- $\text{Sen}(\alpha+\beta) = GD$

  $\text{Sen}(\alpha+\beta) = GH + HD$

  $\text{Sen}(\alpha+\beta) = FE + HD$

  $\mathbf{Sen(\alpha+\beta) = Sen\alpha Cos\beta + Sen\beta Cos\alpha}$

- $\text{Cos}(\alpha+\beta) = ID = OG$

  $\text{Cos}(\alpha+\beta) = OF - GF$

  $\text{Cos}(\alpha+\beta) = OF - HE$

  $\mathbf{Cos(\alpha+\beta) = Cos\alpha Cos\beta - Sen\alpha Sen\beta}$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

$$\left.\begin{aligned} \mathrm{Sen}(\alpha+\beta) &= \mathrm{Sen}\alpha.\mathrm{Cos}\beta + \mathrm{Sen}\beta.\mathrm{Cos}\alpha \\ \mathrm{Cos}(\alpha+\beta) &= \mathrm{Cos}\alpha.\mathrm{Cos}\beta - \mathrm{Sen}\alpha.\mathrm{Sen}\beta \end{aligned}\right\} \div$$

$$\mathrm{Tan}(\alpha+\beta) = \frac{\dfrac{\mathrm{Sen}\alpha\mathrm{Cos}\beta + \mathrm{Sen}\beta\mathrm{Cos}\alpha}{\mathrm{Cos}\alpha\mathrm{Cos}\beta}}{\dfrac{\mathrm{Cos}\alpha\mathrm{Cos}\beta - \mathrm{Sen}\alpha\mathrm{Sen}\beta}{\mathrm{Cos}\alpha\mathrm{Cos}\beta}}$$

$$\mathrm{Tan}(\alpha+\beta) = \frac{\dfrac{\mathrm{Sen}\alpha\mathrm{Cos}\beta}{\mathrm{Cos}\alpha\mathrm{Cos}\beta} + \dfrac{\mathrm{Sen}\beta\mathrm{Cos}\alpha}{\mathrm{Cos}\alpha\mathrm{Cos}\beta}}{\dfrac{\mathrm{Cos}\alpha\mathrm{Cos}\beta}{\mathrm{Cos}\alpha\mathrm{Cos}\beta} - \dfrac{\mathrm{Sen}\alpha\mathrm{Sen}\beta}{\mathrm{Cos}\alpha\mathrm{Cos}\beta}}$$

$$\mathbf{Tan(\alpha+\beta) = \frac{Tan\alpha + Tan\beta}{1 - Tan\alpha Tan\beta}}$$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

1. SUMA DE ÁNGULOS:

- $\text{Sen}(x + y) = \text{Senx}.\text{Cosy} + \text{Seny}.\text{Cosx}$
- $\text{Cos}(x + y) = \text{Cosx}.\text{Cosy} - \text{Senx}.\text{Seny}$
- $\text{Tan}(x + y) = \dfrac{\text{Tanx} + \text{Tany}}{1 - \text{Tanx}.\text{Tany}}$

2. DIFERENCIA DE ÁNGULOS:

- $\mathrm{Sen}(x - y) = \mathrm{Senx.Cosy} - \mathrm{Seny.Cosx}$
- $\mathrm{Cos}(x - y) = \mathrm{Cosx.Cosy} + \mathrm{Senx.Seny}$
- $\mathrm{Tan}(x - y) = \dfrac{\mathrm{Tanx} - \mathrm{Tany}}{1 + \mathrm{Tanx.Tany}}$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

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- $\text{Tan}(x - y) = \dfrac{\text{Tanx} - \text{Tany}}{1 + \text{Tanx}.\text{Tany}}$

3. IDENTIDADES AUXILIARES:

a) $\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x - \text{Sen}^2y$

Demostración:

$$\text{Sen}(x+y).\text{Sen}(x-y) = \underbrace{(\text{Senx.Cosy} + \text{Seny.Cosx})}_{(a+b)} . \underbrace{(\text{Senx.Cosy} - \text{Seny.Cosx})}_{(a-b)} = a^2 - b^2$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = (\text{Senx.Cosy})^2 - (\text{Seny.Cosx})^2$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x.\text{Cos}^2y - \text{Sen}^2y.\text{Cos}^2x$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x.(1 - \text{Sen}^2y) - \text{Sen}^2y.(1 - \text{Sen}^2x)$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x - \cancel{\text{Sen}^2x.\text{Sen}^2y} - \text{Sen}^2y + \cancel{\text{Sen}^2y.\text{Sen}^2x}$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x - \text{Sen}^2y$$

3. IDENTIDADES AUXILIARES:

b) $\mathrm{Cos}(x+y).\mathrm{Cos}(x-y) = \mathrm{Cos}^2x - \mathrm{Sen}^2y$

Demostración:

$$\mathrm{Cos}(x+y).\mathrm{Cos}(x-y) = \underbrace{(\mathrm{Cos}x.\mathrm{Cos}y - \mathrm{Sen}x.\mathrm{Sen}y)}_{(a-b)}.\underbrace{(\mathrm{Cos}x.\mathrm{Cos}y + \mathrm{Sen}x.\mathrm{Sen}y)}_{(a+b)} = a^2 - b^2$$

$$\mathrm{Cos}(x+y).\mathrm{Cos}(x-y) = (\mathrm{Cos}x.\mathrm{Cos}y)^2 - (\mathrm{Sen}x.\mathrm{Sen}y)^2$$

$$\mathrm{Cos}(x+y).\mathrm{Cos}(x-y) = \mathrm{Cos}^2x.\mathrm{Cos}^2y - \mathrm{Sen}^2x.\mathrm{Sen}^2y$$

$$\mathrm{Cos}(x+y).\mathrm{Cos}(x-y) = \mathrm{Cos}^2x.(1-\mathrm{Sen}^2y) - \mathrm{Sen}^2y.(1-\mathrm{Cos}^2x)$$

$$\mathrm{Cos}(x+y).\mathrm{Cos}(x-y) = \mathrm{Cos}^2x - \cancel{\mathrm{Cos}^2x.\mathrm{Sen}^2y} - \mathrm{Sen}^2y + \cancel{\mathrm{Sen}^2y.\mathrm{Cos}^2x}$$

$$\mathrm{Cos}(x+y).\mathrm{Cos}(x-y) = \mathrm{Cos}^2x - \mathrm{Sen}^2y$$

3. IDENTIDADES AUXILIARES:

c) $\mathrm{Tan}(x+y).\mathrm{Tan}(x-y) = \dfrac{\mathrm{Tan}^2x - \mathrm{Tan}^2y}{1 - \mathrm{Tan}^2x\mathrm{Tan}^2y}$

Demostración:

$$\mathrm{Tan}(x+y).\mathrm{Tan}(x-y) = \left(\frac{\mathrm{Tan}x + \mathrm{Tan}y}{1 - \mathrm{Tan}x.\mathrm{Tan}y}\right)\left(\frac{\mathrm{Tan}x - \mathrm{Tan}y}{1 + \mathrm{Tan}x.\mathrm{Tan}y}\right)$$

$$\mathrm{Tan}(x+y).\mathrm{Tan}(x-y) = \frac{\mathrm{Tan}^2x - \mathrm{Tan}^2y}{1 - \mathrm{Tan}^2x\mathrm{Tan}^2y}$$

3. IDENTIDADES AUXILIARES:

d) $\text{Tanx} \pm \text{Tany} = \dfrac{\text{Sen}(x \pm y)}{\text{CosxCosy}}$

Demostración:

$$\text{Tanx} + \text{Tany} = \frac{\text{Senx}}{\text{Cosx}} + \frac{\text{Seny}}{\text{Cosy}}$$

$$\text{Tanx} + \text{Tany} = \frac{\text{Senx.Cosy} + \text{Seny.Cosx}}{\text{Cosx.Cosy}}$$

$$\text{Tanx} + \text{Tany} = \frac{\text{Sen}(x + y)}{\text{CosxCosy}}$$

3. IDENTIDADES AUXILIARES:

e) $\text{Cotx} \pm \text{Coty} = \dfrac{\text{Sen}(y \pm x)}{\text{SenySenx}}$

Demostración:

$$\text{Cotx} - \text{Coty} = \frac{\text{Cosx}}{\text{Senx}} - \frac{\text{Cosy}}{\text{Seny}}$$

$$\text{Cotx} - \text{Coty} = \frac{\text{Seny.Cosx} - \text{Senx.Cosy}}{\text{Senx.Seny}}$$

$$\text{Cotx} - \text{Coty} = \frac{\text{Sen}(y - x)}{\text{SenxSeny}}$$

3. IDENTIDADES AUXILIARES:

f) $\mathrm{Tan}x \pm \mathrm{Tan}y \pm \mathrm{Tan}(x \pm y).\mathrm{Tan}x.\mathrm{Tan}y = \mathrm{Tan}(x \pm y)$

Demostración:

$$\mathrm{Tan}(x+y) = \frac{\mathrm{Tan}x + \mathrm{Tan}y}{1 - \mathrm{Tan}x.\mathrm{Tan}y}$$

$$\mathrm{Tan}(x+y)(1 - \mathrm{Tan}x.\mathrm{Tan}y) = \mathrm{Tan}x + \mathrm{Tan}y$$

$$\mathrm{Tan}(x+y) - \mathrm{Tan}(x+y).\mathrm{Tan}x.\mathrm{Tan}y = \mathrm{Tan}x + \mathrm{Tan}y$$

$$\mathrm{Tan}(x+y) = \mathrm{Tan}x + \mathrm{Tan}y + \mathrm{Tan}(x+y).\mathrm{Tan}x.\mathrm{Tan}y$$

$$\mathrm{Tan}(x+y) = \mathrm{Tan}x + \mathrm{Tan}y + \mathrm{Tan}(x+y).\mathrm{Tan}x.\mathrm{Tan}y$$

3. IDENTIDADES AUXILIARES:

a) $\mathrm{Sen}(x+y)\mathrm{Sen}(x-y) = \mathrm{Sen}^2x - \mathrm{Sen}^2y$

b) $\mathrm{Cos}(x+y)\mathrm{Cos}(x-y) = \mathrm{Cos}^2x - \mathrm{Sen}^2y$

c) $\mathrm{Tan}(x+y)\mathrm{Tan}(x-y) = \dfrac{\mathrm{Tan}^2x - \mathrm{Tan}^2y}{1 - \mathrm{Tan}^2x\mathrm{Tan}^2y}$

d) $\mathrm{Tan}x \pm \mathrm{Tan}y = \dfrac{\mathrm{Sen}(x \pm y)}{\mathrm{Cos}x\mathrm{Cos}y}$

e) $\mathrm{Cot}x \pm \mathrm{Cot}y = \dfrac{\mathrm{Sen}(y \pm x)}{\mathrm{Sen}y\mathrm{Sen}x}$

f) $\mathrm{Tan}x \pm \mathrm{Tan}y \pm \mathrm{Tan}(x \pm y).\mathrm{Tan}x.\mathrm{Tan}y = \mathrm{Tan}(x \pm y)$

4. PROPIEDAD:

I) $$\frac{E}{\sqrt{a^2+b^2}} = \frac{aSenx}{\sqrt{a^2+b^2}} \pm \frac{bCosx}{\sqrt{a^2+b^2}}$$

$$\frac{E}{\sqrt{a^2+b^2}} = Senx\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{Cos\varphi} \pm Cosx\underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{Sen\varphi}$$

$$\frac{E}{\sqrt{a^2+b^2}} = Senx.Cos\varphi \pm Cosx.Sen\varphi$$

$$\frac{E}{\sqrt{a^2+b^2}} = Sen(x \pm \varphi)$$

$$E = \sqrt{a^2+b^2}Sen(x \pm \varphi)$$

$$\left.\frac{b}{\sqrt{a^2+b^2}}\right\} Sen\varphi$$

$$\left.\frac{a}{\sqrt{a^2+b^2}}\right\} Cos\varphi$$

$$\rightarrow Tan\varphi = \frac{b}{a}$$

4. PROPIEDAD:

II) Sea: $E = a\text{Sen}x + b\text{Cos}x, x \in \mathbb{R}$

$$E = \sqrt{a^2 + b^2}\text{Sen}(x + \varphi)$$

Si: $x \in \mathbb{R} \rightarrow (x + \varphi) \in \mathbb{R}$

$$-1 \leq \text{Sen}(x + \varphi) \leq 1$$

$\times\left(\sqrt{a^2 + b^2}\right)$:

$$-\sqrt{a^2 + b^2} \leq \underbrace{\sqrt{a^2 + b^2}\text{Sen}(x + \varphi)}_{E} \leq \sqrt{a^2 + b^2}$$

$$\underbrace{-\sqrt{a^2 + b^2}}_{E_{mín}} \leq E \leq \underbrace{\sqrt{a^2 + b^2}}_{E_{máx}}$$

4. PROPIEDADES:

I) $\mathrm{aSenx} \pm \mathrm{bCosx} = \sqrt{a^2 + b^2}\,\mathrm{Sen}(x \pm \varphi), a \text{ y } b \in \mathbb{R}$

Donde: $\mathrm{Tan}\varphi = \dfrac{b}{a}$

II) Sea: $E = \mathrm{aSenx} + \mathrm{bCosx}, x \in \mathbb{R}$

$$\underbrace{-\sqrt{a^2 + b^2}}_{\text{mín}} \leq \mathrm{aSenx} + \mathrm{bCosx} \leq \underbrace{\sqrt{a^2 + b^2}}_{\text{máx}}$$

5. PROPIEDADES PARA 3 ÁNGULOS:

I) Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$x + y = n\pi - z$$

$$\mathrm{Tan}(x + y) = \mathrm{Tan}(\overbrace{n\pi - z}^{\in \mathrm{IIC} \vee \mathrm{IVC}})$$

$$\frac{\mathrm{Tan}x + \mathrm{Tan}y}{1 - \mathrm{Tan}x.\mathrm{Tan}y} = -\mathrm{Tan}z$$

$$\mathrm{Tan}x + \mathrm{Tan}y = -\mathrm{Tan}z + \mathrm{Tan}x.\mathrm{Tan}y.\mathrm{Tan}z$$

$$\mathrm{Tan}x + \mathrm{Tan}y + \mathrm{Tan}z = \mathrm{Tan}x.\mathrm{Tan}y.\mathrm{Tan}z$$

5. PROPIEDADES PARA 3 ÁNGULOS:

II) Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$\mathrm{Tan}x + \mathrm{Tan}y + \mathrm{Tan}z = \mathrm{Tan}x.\mathrm{Tan}y.\mathrm{Tan}z$$

$$\frac{\cancel{\mathrm{Tan}x}}{\cancel{\mathrm{Tan}x}.\mathrm{Tan}y.\mathrm{Tan}z} + \frac{\cancel{\mathrm{Tan}y}}{\mathrm{Tan}x.\cancel{\mathrm{Tan}y}.\mathrm{Tan}z} + \frac{\cancel{\mathrm{Tan}z}}{\mathrm{Tan}x.\mathrm{Tan}y.\cancel{\mathrm{Tan}z}} = \frac{\cancel{\mathrm{Tan}x.\mathrm{Tan}y.\mathrm{Tan}z}}{\cancel{\mathrm{Tan}x.\mathrm{Tan}y.\mathrm{Tan}z}}$$

$$\mathrm{Cot}x.\mathrm{Cot}y + \mathrm{Cot}x.\mathrm{Cot}z + \mathrm{Cot}y.\mathrm{Cot}z = 1$$

5. PROPIEDADES PARA 3 ÁNGULOS:

- Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$\mathrm{Tan}x + \mathrm{Tan}y + \mathrm{Tan}z = \mathrm{Tan}x.\mathrm{Tan}y.\mathrm{Tan}z$$

$$\mathrm{Cot}x.\mathrm{Cot}y + \mathrm{Cot}x.\mathrm{Cot}z + \mathrm{Cot}y.\mathrm{Cot}z = 1$$

5. PROPIEDADES PARA 3 ÁNGULOS:

I) Si: $x + y + z = (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$

$$x + y = (2n+1)\frac{\pi}{2} - z$$

$$\mathrm{Cot}(x+y) = \mathrm{Cot}\left(\overbrace{(2n+1)\frac{\pi}{2} - z}^{\in \mathrm{IC} \ \vee \ \mathrm{IIIC}}\right)$$

$$\frac{1 - \mathrm{Tan}x.\mathrm{Tan}y}{\mathrm{Tan}x + \mathrm{Tan}y} = \mathrm{Tan}z$$

$$1 - \mathrm{Tan}x.\mathrm{Tan}y = \mathrm{Tan}x.\mathrm{Tan}z + \mathrm{Tan}y.\mathrm{Tan}z$$

$$\mathrm{Tan}x.\mathrm{Tan}y + \mathrm{Tan}x.\mathrm{Tan}z + \mathrm{Tan}y.\mathrm{Tan}z = 1$$

5. PROPIEDADES PARA 3 ÁNGULOS:

II) Si: $x + y + z = (2n + 1)\frac{\pi}{2}; n \in \mathbb{Z}$

$$\mathrm{Tanx.Tany} + \mathrm{Tanx.Tanz} + \mathrm{Tany.Tanz} = 1$$

$$\frac{\cancel{\mathrm{Tanx}}.\cancel{\mathrm{Tany}}}{\cancel{\mathrm{Tanx}}.\cancel{\mathrm{Tany}}.\mathrm{Tanz}} + \frac{\cancel{\mathrm{Tanx}}.\cancel{\mathrm{Tanz}}}{\cancel{\mathrm{Tanx}}.\mathrm{Tany}.\cancel{\mathrm{Tanz}}} + \frac{\cancel{\mathrm{Tany}}.\cancel{\mathrm{Tanz}}}{\mathrm{Tanx}.\cancel{\mathrm{Tany}}.\cancel{\mathrm{Tanz}}} = \frac{1}{\mathrm{Tanx.Tany.Tanz}}$$

$$\mathrm{Cotx} + \mathrm{Coty} + \mathrm{Cotz} = \mathrm{Cotx.Coty.Cotz}$$

5. PROPIEDADES PARA 3 ÁNGULOS :

- Si: $x + y + z = (2n + 1)\frac{\pi}{2}; n \in \mathbb{Z}$

$$\mathrm{Cotx} + \mathrm{Coty} + \mathrm{Cotz} = \mathrm{Cotx}.\mathrm{Coty}.\mathrm{Cotz}$$

$$\mathrm{Tanx}.\mathrm{Tany} + \mathrm{Tanx}.\mathrm{Tanz} + \mathrm{Tany}.\mathrm{Tanz} = 1$$

5. PROPIEDADES PARA 3 ÁNGULOS:

- Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$\mathrm{Tan}x + \mathrm{Tan}y + \mathrm{Tan}z = \mathrm{Tan}x.\mathrm{Tan}y.\mathrm{Tan}z$$

$$\mathrm{Cot}x.\mathrm{Cot}y + \mathrm{Cot}x.\mathrm{Cot}z + \mathrm{Cot}y.\mathrm{Cot}z = 1$$

- Si: $x + y + z = (2n + 1)\frac{\pi}{2}; n \in \mathbb{Z}$

$$\mathrm{Cot}x + \mathrm{Cot}y + \mathrm{Cot}z = \mathrm{Cot}x.\mathrm{Cot}y.\mathrm{Cot}z$$

$$\mathrm{Tan}x.\mathrm{Tan}y + \mathrm{Tan}x.\mathrm{Tan}z + \mathrm{Tan}y.\mathrm{Tan}z = 1$$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

IDENTIDADES PARA EL ÁNGULO COMPUESTO

1. Reducir: $E = \dfrac{\text{Sen}\theta\text{Cos}(\alpha-\beta) - \text{Sen}\alpha\text{Cos}(\theta-\beta)}{\text{Cos}\beta\text{Sen}(\theta-\alpha)}$

Resolución:

$$E = \frac{\text{Sen}\theta(\text{Cos}\alpha\text{Cos}\beta + \text{Sen}\alpha\text{Sen}\beta) - \text{Sen}\alpha(\text{Cos}\theta\text{Cos}\beta + \text{Sen}\theta\text{Sen}\beta)}{\text{Cos}\beta(\text{Sen}\theta\text{Cos}\alpha - \text{Sen}\alpha\text{Cos}\theta)}$$

$$E = \frac{\text{Sen}\theta\text{Cos}\alpha\text{Cos}\beta + \cancel{\text{Sen}\theta\text{Sen}\alpha\text{Sen}\beta} - \text{Sen}\alpha\text{Cos}\theta\text{Cos}\beta - \cancel{\text{Sen}\alpha\text{Sen}\theta\text{Sen}\beta}}{\text{Cos}\beta\text{Sen}\theta\text{Cos}\alpha - \text{Cos}\beta\text{Sen}\alpha\text{Cos}\theta}$$

$$\therefore \mathbf{E = 1}$$

CLAVE: B

IDENTIDADES PARA EL ÁNGULO COMPUESTO

2. Calcular el valor de: $P = \dfrac{\text{Tan}50° - 2\text{Tan}10°}{\text{Tan}40°}$

Resolución:

$$10° = 50° - 40°$$

$$\text{Tan}10° = \text{Tan}(50° - 40°)$$

$$\text{Tan}10° = \frac{\text{Tan}50° - \text{Tan}40°}{1 + \text{Tan}50°\text{Tan}40°}$$

$$\text{Tan}10° = \frac{\text{Tan}50° - \text{Tan}40°}{1 + \underbrace{\text{Tan}50°\text{Cot}50°}_{1}}$$

$$\text{Tan}10° = \frac{\text{Tan}50° - \text{Tan}40°}{2}$$

$$2\text{Tan}10° = \text{Tan}50° - \text{Tan}40°$$

$$\text{Tan}40° = \text{Tan}50° - 2\text{Tan}10°$$

$$P = \frac{\text{Tan}50° - 2\text{Tan}10°}{\text{Tan}40°}$$

$$\therefore \mathbf{P = 1}$$

CLAVE: B

3. Si se cumple que: $4\text{Senx}\text{Tan}(x-y)+3\text{Secx}=4\text{Sen}^2\text{xSecx}$, calcular el valor de "TanxTany"

Resolución:

$$4\text{Senx}\text{Tan}(x-y)+3\text{Secx}=4\text{Sen}^2\text{xSecx}$$

$$4\text{Senx}\text{Tan}(x-y)+3\times\frac{1}{\text{Cosx}}=4\text{Senx}\text{Senx}\times\frac{1}{\text{Cosx}}$$

$$\frac{3}{\text{Cosx}}=4\text{Senx}\text{Tanx}-4\text{Senx}\text{Tan}(x-y)$$

$$\frac{3}{\text{Cosx}}=4\text{Senx}\big(\text{Tanx}-\text{Tan}(x-y)\big)$$

$$\frac{3}{\cancel{\text{Cosx}}}=4\text{Senx}\left(\frac{\text{Seny}}{\cancel{\text{Cosx}}.\,\text{Cos}(x-y)}\right)$$

$$3\text{Cos}(x-y)=4\text{Senx}\text{Seny}$$

$$3\text{Cosx}\text{Cosy}+3\text{Senx}\text{Seny}=4\text{Senx}\text{Seny}$$

$$\text{Senx}\text{Seny}=3\text{Cosx}\text{Cosy}$$

$$\therefore\ \mathbf{TanxTany=3}$$

CLAVE: D

4. Si se cumple que: $\mathrm{TanA} - \mathrm{Tan}(\mathrm{A}-\mathrm{B}) = \dfrac{\mathrm{Sen}^2\mathrm{x}}{\mathrm{SenACosA}}$, calcular $\mathrm{Tan}^2\mathrm{x}$

Resolución:

$$\mathrm{TanA} - \mathrm{Tan}(\mathrm{A}-\mathrm{B}) = \frac{\mathrm{Sen}^2\mathrm{x}}{\mathrm{SenACosA}}$$

$$\frac{\mathrm{SenB}}{\cancel{\mathrm{CosA}}\mathrm{Cos}(\mathrm{A}-\mathrm{B})} = \frac{\mathrm{Sen}^2\mathrm{x}}{\mathrm{SenA}\cancel{\mathrm{CosA}}}$$

$$\frac{\mathrm{SenASenB}}{\mathrm{Cos}(\mathrm{A}-\mathrm{B})} = \mathrm{Sen}^2\mathrm{x}$$

$$\frac{\mathrm{CosACosB} + \mathrm{SenASenB}}{\mathrm{SenASenB}} = \mathrm{Csc}^2\mathrm{x}$$

$$\frac{\mathrm{CosACosB}}{\mathrm{SenASenB}} + \cancel{1} = \mathrm{Cot}^2\mathrm{x} + \cancel{1}$$

$$\therefore \mathbf{Tan^2x = TanATanB}$$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

5. Si: $\alpha - \beta = 60°$, calcular el máximo valor de: $M = 2Sen(x + \alpha) + 3Sen(x + \beta)$

Resolución:

$$M = 2Sen(x + \alpha) + 3Sen(x + \beta)$$

$$M = 2Sen(\underbrace{x + (60° + \beta)}_{\theta}) + 3Sen(\underbrace{x + \beta}_{\theta})$$

$$M = 2Sen60°Cos\theta + 2Sen\theta Cos60° + 3Sen\theta$$

$$M = 2 \times \frac{\sqrt{3}}{2} Cos\theta + 2Sen\theta \times \frac{1}{2} + 3Sen\theta$$

$$M = 4Sen\theta + \sqrt{3}Cos\theta$$

$$-\sqrt{19} \leq 4Sen\theta + \sqrt{3}Cos\theta \leq \sqrt{19}$$

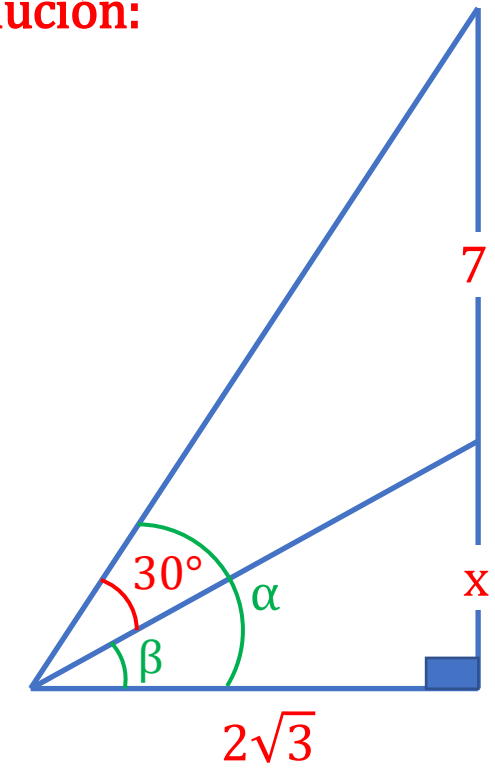
$$\therefore \mathbf{M_{máx} = \sqrt{19}}$$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

6. De la figura, calcular el valor de x

Resolución:



$$30° = \alpha - \beta$$

$$\text{Tan}30° = \text{Tan}(\alpha - \beta)$$

$$\frac{1}{\cancel{\sqrt{3}}} = \frac{\dfrac{x+7}{2\cancel{\sqrt{3}}} - \dfrac{x}{2\cancel{\sqrt{3}}}}{1 + \dfrac{x+7}{2\sqrt{3}} \times \dfrac{x}{2\sqrt{3}}}$$

$$1 + \frac{x^2 + 7x}{12} = \frac{7}{2}$$

$$\frac{x^2 + 7x}{12} = \frac{5}{2}$$

$$x^2 + 7x - 30 = 0$$

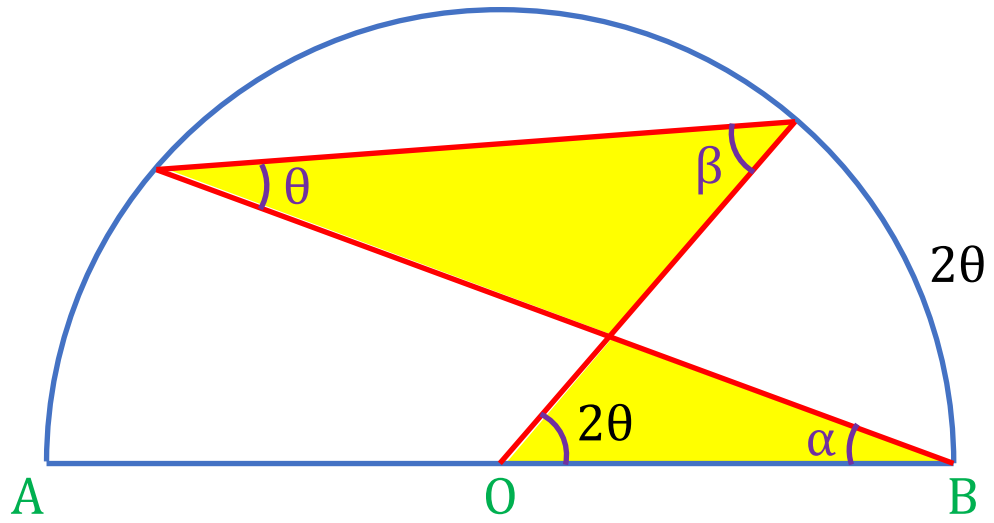
$$\begin{matrix} x & & 10 \\ x & & -3 \end{matrix}$$

$$\therefore \mathbf{x = 3}$$

CLAVE: D

IDENTIDADES PARA EL ÁNGULO COMPUESTO

7. Del gráfico, calcular "Tanθ", si Tanα = 2/3 y Tanβ = 3/2; (O: Centro de la semicircunferencia)



Resolución:

$$2\theta + \alpha = \theta + \beta$$

$$\theta = \beta - \alpha$$

$$\text{Tan}\theta = \text{Tan}(\beta - \alpha)$$

$$\text{Tan}\theta = \frac{\frac{3}{2} - \frac{2}{3}}{1 + \frac{3}{2} \times \frac{2}{3}}$$

$$\text{Tan}\theta = \frac{\frac{5}{6}}{1 + 1}$$

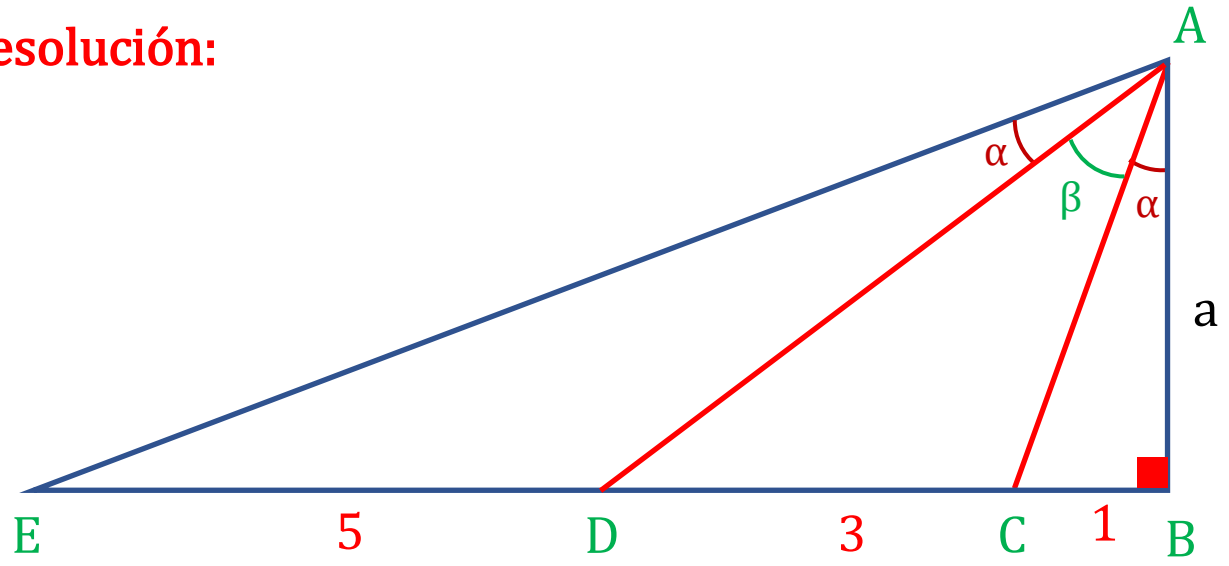
$$\therefore \mathbf{Tan\theta = \frac{5}{12}}$$

CLAVE: D

IDENTIDADES PARA EL ÁNGULO COMPUESTO

8. Del gráfico mostrado, calcular, "Tanβ"

Resolución:



$$\mathrm{Tan}\alpha = \frac{1}{a} \qquad \mathrm{Tan}(\alpha+\beta) = \frac{4}{a} \qquad \mathrm{Tan}(\alpha+(\alpha+\beta)) = \frac{9}{a}$$

$$\alpha = \frac{37°}{2} \qquad (\alpha+\beta) = 53°$$

$$\frac{\mathrm{Tan}\alpha + \mathrm{Tan}(\alpha+\beta)}{1-\mathrm{Tan}\alpha\mathrm{Tan}(\alpha+\beta)} = \frac{9}{a}$$

$$\frac{\frac{4}{a}+\frac{1}{a}}{1-\frac{4}{a}\times\frac{1}{a}} = \frac{9}{a}$$

$$5 = 9\left(1-\frac{4}{a^2}\right)$$

$$a = 3$$

$$(\beta) = 53° - \frac{37°}{2}$$

$$\mathrm{Tan}(\beta) = \frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}\times\frac{1}{3}}$$

$$\therefore \mathbf{Tan(\beta) = \frac{9}{13}}$$

CLAVE: E

IDENTIDADES PARA EL ÁNGULO COMPUESTO

9. En un triángulo ABC, se cumple que: TanA - TanB = TanC ¿A qué es igual? E = TanBTanC

Resolución:

$$A + B + C = 180°$$

$$\text{TanA} + \text{TanB} + \text{TanC} = \text{TanATanBTanC}$$

$$\text{TanA} = \text{TanB} + \text{TanC}$$

$+\text{TanA}$

$$2\text{TanA} = \text{TanA} + \text{TanB} + \text{TanC}$$

$$2\cancel{\text{TanA}} = \cancel{\text{TanA}}\text{TanBTanC}$$

$$\therefore \mathbf{TanBTanC = 2}$$

CLAVE: D

IDENTIDADES PARA EL ÁNGULO COMPUESTO

10. En un triángulo ABC, reducir: $S = \dfrac{\text{Cos}(A-B)}{\text{SenASenB}} + \dfrac{\text{Cos}(B-C)}{\text{SenBSenC}} + \dfrac{\text{Cos}(A-C)}{\text{SenASenC}}$

Resolución:

$$S = \frac{\text{CosACosB} + \text{SenASenB}}{\text{SenASenB}} + \frac{\text{CosBCosC} + \text{SenBSenC}}{\text{SenBSenC}} + \frac{\text{CosACosC} + \text{SenASenC}}{\text{SenASenC}}$$

$$S = \frac{\text{CosACosB}}{\text{SenASenB}} + 1 + \frac{\text{CosBCosC}}{\text{SenBSenC}} + 1 + \frac{\text{CosACosC}}{\text{SenASenC}} + 1$$

$$S = 3 + \underbrace{\text{CotACotB} + \text{CotBCotC} + \text{CotACotC}}_{1}$$

$\therefore \mathbf{S = 4}$

CLAVE: D

IDENTIDADES PARA EL ÁNGULO COMPUESTO

11. Si Sen(a+b)=SenbCosa, donde "a" es un ángulo no cuadrantal, calcular el valor de la expresión : $W = \dfrac{\mathrm{Cos}^2(a+b)}{(\mathrm{Sen}a + \mathrm{Cos}b)^2}$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

11. Si Sen(a+b)=SenbCosa, donde "a" es un ángulo no cuadrantal, calcular el valor de la expresión : $W = \dfrac{\text{Cos}^2(a+b)}{(\text{Sena} + \text{Cosb})^2}$

Resolución:

$$\text{Sen}(a+b) = \text{SenbCosa}$$

$$\text{SenaCosb} + \cancel{\text{SenbCosa}} = \cancel{\text{SenbCosa}}$$

$$\underbrace{\text{Sena}}_{\neq 0}\text{Cosb} = 0$$

$$\rightarrow \text{Cosb} = 0$$

$$b = (2n+1)\frac{\pi}{2}$$

$$W = \frac{\text{Cos}^2(a+b)}{(\text{Sena} + \text{Cosb})^2}$$

$$W = \frac{\text{Cos}^2\left[(2n+1)\frac{\pi}{2} + a\right]}{\left(\text{Sena} + \text{Cos}(2n+1)\frac{\pi}{2}\right)^2}$$

$$W = \frac{(\pm\text{Sena})^2}{(\text{Sena} + 0)^2}$$

$$W = \frac{\text{Sen}^2 a}{\text{Sen}^2 a}$$

$$\therefore \mathbf{W = 1}$$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

12. Si Cos(a-b) = 2Sen(x+a)Sen(x+b), calcular Tan(x+a)Tan(x+b)

CLAVE: B

13. Si se tiene que : $\alpha+\theta = 2\pi$ y $b\text{Sen}\alpha = a\text{Cos}\theta$, calcular "$\text{Cot}(\alpha-\theta)$"

CLAVE: B

14. Calcular el valor de la expresión :$P = (2\text{Cos}50° + \text{Sen}20°)\ \text{Sec}20°$

CLAVE: D

15. Si se cumple que : $\mathrm{Tan}(\theta+\alpha)=\frac{3}{2}\mathrm{Tan}(a)$, $\mathrm{Tan}(\theta-\alpha)=\frac{1}{2}\mathrm{Tan}(a)$ y $\mathrm{Tan}2\theta=3\mathrm{Tan}(a)$;

calcular el valor de $\mathrm{Tan}(a)$, si $a \in \mathrm{IIC}$

CLAVE: B

16. Si $\text{Tan}(a+12°)=2$, $\text{Tan}(b+15°)=3$ y $\text{Tan}(c+18°)=4$, calcular el valor de $\text{Cot}(a+b+c)$

CLAVE: E

IDENTIDADES PARA EL ÁNGULO COMPUESTO

17. Calcular :

$$E = \frac{(\sqrt{3}+1)\text{Tan}\frac{\pi}{12}+1}{\text{Tan}\frac{\pi}{24}+\text{Tan}\frac{5\pi}{24}+\text{Tan}\frac{\pi}{24}.\text{Tan}\frac{5\pi}{24}}$$

CLAVE: B

18. Sabiendo que: $\mathrm{Tan}x\mathrm{Tan}2x + \mathrm{Tan}2x\mathrm{Tan}3x + \mathrm{Tan}3x\mathrm{Tan}4x + \mathrm{Tan}4x\mathrm{Tan}5x = 14$, calcular el valor de "$\mathrm{Tan}5x\mathrm{Cot}x$"

CLAVE: D

19. Siendo x, y, z, tres ángulos, tales que $x + y + z = 90°$, además $\mathrm{Sec}^2 x + \mathrm{Sec}^2 y + \mathrm{Sec}^2 z = \mathrm{Sec}^2\theta$; $\theta \in \mathrm{IIC}$
Calcular el valor de: $M = \mathrm{Tan}x + \mathrm{Tan}y + \mathrm{Tan}z + \mathrm{Tan}\theta$

CLAVE: C

20. En un triángulo rectángulo ABC, se cumple: $\operatorname{Tan}\frac{A}{2}+\operatorname{Tan}\frac{B}{2}+\operatorname{Tan}\frac{C}{2}=1{,}96$, calcular: $E=\operatorname{Cot}\frac{A}{2}+\operatorname{Cot}\frac{B}{2}+\operatorname{Cot}\frac{C}{2}$

CLAVE: C

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS